FIRST EVALUATION AND EXPERIMENTAL RESULTS ON THE DETERMINATION OF UNCERTAINTIES IN [UTC – UTC (k)]

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Abstract

In this work, we present a preliminary study of the uncertainty of [UTC – UTC (k)]. In the first part of the paper, we consider an analytical solution considering the law of the propagation of uncertainty. In the second part, we verify the analytical results numerically, using the software used for the generation of UTC.

INTRODUCTION

Coordinated Universal Time (UTC), the worldwide time standard, is disseminated monthly through the publication of [*UTC* – *UTC* (*k*)] in *Circular T* by the Bureau International des Poids et Mesures (BIPM). Until now, these values have been published without their uncertainties, but the rules of the CIPM Mutual Recognition Arrangement (CIPM MRA) require the evaluation of this uncertainty. This paper reports the first steps towards computing of these uncertainties.

UTC is derived from International Atomic Time (TAI) by the addition of leap seconds, while TAI is derived from the Free Atomic Timescale (EAL) through the addition of pre-announced frequency steers determined by comparison with a weighted set of primary frequency standards. EAL is a worldwide weighted average of a large number of free-running, effectively uncalibrated, frequency standards **[1-3]**. The uncertainty in the determination of EAL, TAI, and UTC, as steps in the realization of Terrestrial Time, is affected by three major elements: clock variations, the means of comparisons of remote clocks (time transfer), and the time-scale algorithm. The uncertainties of time transfer are particularly significant over averaging times of up to a few tens of days, and also influence the uncertainty of the access of the participating laboratories *k* to UTC, in other words the uncertainty of [*UTC* – *UTC* (*k*)].

In this work we present a preliminary study of the determination of the uncertainties of [*UTC* – *UTC* (*k*)]. In the first part of the paper, we develop an analytical solution using the law of the propagation of uncertainty. In the second part, we carry out a numerical verification of the analytical results using the software used for the generation of UTC.

1 THE ALGORITHMS OF EAL, TAI, AND UTC AND WORKING HYPOTHESIS

In order to obtain the uncertainty of *UTC – UTC (k),* we consider the general equation of the free atomic time scale (EAL). EAL is defined using the ALGOS algorithm **[1-3]** as:

$$EAL(t) = \sum_{i=1}^{N} w_i \left[h_i(t) + h_i'(t)\right] \tag{1}$$

where N is the number of the atomic clocks, w_i the weight of the clocks, $h_i(t)$ is the reading of clock H_i at time t, and $h_i'(t)$ is the prediction of the reading of clock H_i to guarantee the continuity of the time scale. The weight attributed to a given clock reflects its long-term stability, since the objective is to obtain a weighted average that is more stable in the long term than any of the contributing elements **[4, 5]**. The weights of the clocks obey the relation:

$$\sum_{i=1}^{N} w_i = 1 \tag{2}$$

Subtracting the same quantity from both sides of (1), we obtain:

$$EAL(t) - \sum_{i=1}^{N} w_i h_i(t) = \sum_{i=1}^{N} w_i \left[h_i(t) + h_i'(t)\right] - \sum_{i=1}^{N} w_i h_i(t).$$

Using (2) and rearranging we obtain:

$$\sum_{i=1}^{N} w_i \left(EAL(t) - h_i(t)\right) = \sum_{i=1}^{N} w_i h_i'(t). \tag{3}$$

Setting

$$x_i(t) = EAL(t) - h_i(t), \tag{4}$$

it is clear that equation (3) is of the form:

$$\sum_{i=1}^{N} w_i x_i(t) = \sum_{i=1}^{N} w_i h_i'(t). \tag{5}$$

The software package termed ALGOS is used in the Time Section of the BIPM in order to generate UTC. Weights are determined from the variance of monthly average frequencies, subject to a maximum value **[5]**. The data used by ALGOS take the form of the time differences between readings of clocks, written as:

$$x_{i,j}(t) = h_j(t) - h_i(t). \tag{6}$$

Equation (5) in conjunction with the N – 1 equations (6), results in a system with N equations and N unknowns:

$$\begin{cases} \sum_{i=1}^{N} w_i x_i(t) = \sum_{i=1}^{N} w_i h_i'(t) \\ x_i(t) - x_j(t) = x_{i,j}(t) \end{cases} \tag{7}$$

The solution is:

$$x_j(t) = EAL(t) - h_j = \sum_{i=1}^{N} w_i \left[h_i'(t) - x_{i,j}(t)\right]. \tag{8}$$

If we choose a particular clock H_j, we can see that the difference between that clock and EAL (8) depends on weights, clock prediction, and measured clock differences. The clock H_j may also represent a *UTC (j)* time scale; therefore, we may also interpret $x_j(t)$ as:

$$x_j(t) = EAL\,(t) - UTC\,(j)$$

The predictions and the weights are fixed by appropriate algorithms based on the past clock behavior and in (8) they can be considered as time-varying deterministic parameters. Suboptimal estimation of these parameters would affect the uncertainty of *TAI* as realization of the Terrestrial Time (*TT*), but they do not affect the knowledge of the difference between *EAL* and clock $H_{j;}$ the measures $x_{i,j}$ are thus the only contributors to the uncertainties in x_j. In particular, we consider negligible the contribution of the uncertainty given by measures of clocks located inside the same laboratory.

Moreover, [*UTC* – *EAL*] depends only on pre-determined leap seconds and frequency steers that do not add uncertainty. The uncertainties of [*UTC* – *UTC* (*k*)] are therefore close to the uncertainties of [*TAI* – *UTC* (*k*)] and [*EAL* – *UTC* (*k*)].

We conclude considering the uncertainties of the links among laboratories as the only source of the uncertainty of *UTC – UTC (k)*.

1.1 THE LAW OF PROPAGATION OF UNCERTAINTY

According to **[6]**, the uncertainty in the $x_j(t)$ can be found using the law of propagation of uncertainty. Let's indicate with y a generic quantity indirectly measured by means of direct measurements of the input quantity x_i,:

$$y = f(x_1, x_2, ..., x_M).$$

The expression of the law of the propagation of uncertainty **[6]** is given by:

$$u_y^2 = \sum_{i=1}^{M} \left(\frac{\partial f}{\partial x_i}\right)^2 u_{x_i}^2 + 2\sum_{i=1}^{M-1} \sum_{k=i+1}^{M} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_k} u_{(x_i, x_k)} \tag{10}$$

where the first term corresponds to the effect of the uncertainties on the input quantities x_i, and the second term accounts for the correlation between them.

In our case, we are interested in evaluating the uncertainty of the quantity x_k = [*EAL* – *UTC* (*k*)] defined in (8), here playing the role of the indirect quantity *y,* and the uncertainty contributions are only due to the measurement noise of the links $x_{i,j}(t)$. Applying (10) to our model (9) yields:

$$u_{x_j}^2 = \sum_{i=1}^{N} \left(\frac{\partial x_j}{\partial x_{i,j}}\right)^2 u_{x_{i,j}}^2 + 2\sum_{i=1}^{N-1} \sum_{k=i+1}^{N} \frac{\partial x_j}{\partial x_{i,j}} \frac{\partial x_j}{\partial x_{k,j}} u_{(x_{i,j}, x_{k,j})} = \sum_{i=1}^{N} w_i^2 u_{x_{i,j}}^2 + 2\sum_{i=1}^{N-1} \sum_{k=i+1}^{N} w_i w_k u_{(x_{i,j}, x_{k,j})} \tag{11}$$

where $u_{x_j}^2 = u_{EAL-h_j}^2$.

The weights of the clocks are available from the BIPM Web site, and the uncertainty of links between the clocks **[7]** are published in *Circular T* (see the BIPM Time Section's FTP server at *www.bipm.org*).

The propagation of uncertainty (11) could as well be expressed by a matrix formulation using a multivariate weighted approach **[11,12]**; here, we present the scalar approach that allows one to clearly understand how the different components contribute to the combined uncertainty.

It can be demonstrated that the ALGOS algorithm would generate the same results if each laboratory's clocks were replaced by a single "equivalent" clock whose reading was the weighted average of the individual clocks and whose weight in EAL was the sum of the individual clock weights. Therefore, we can simplify the computations by summing the weights of the clocks at each lab as follows:

$$W_{Lab_k} = \sum_{i=1}^{N_{Lab_k}} w_i \tag{12}$$

where N_{Lab_k} is the number of clocks of the considered lab k.

Since we neglect the uncertainty of the measurement between clocks located inside the same lab, at any lab we can consider a single equivalent clock used as reference for the time transfer with the other labs. We note that the formalism of equation (11) could still be applied without these simplifying assumptions, as the double summation would account for the 100% correlation of the time transfer noise between clocks in the same lab.

1.2 EXTENSION OF THE COMPUTATION

It is not necessary to apply equation (11) to every laboratory. If the uncertainty for a clock is known, for example considering $x_j = EAL - \mathrm{h}_j$, the evaluation of uncertainty on $x_i = EAL - \mathrm{h}_i$ may be obtained by using the second equation in (7) and applying the property of the variance to obtain:

$$u_{x_i}^2 = u_{x_j}^2 + u_{x_{i,j}}^2 + 2u_{x_j, x_{i,j}} \tag{13}$$

The last term is the covariance between the measures $x_{i,j}$ and the quantity $x_j = EAL - \mathrm{h}_j$. Since in EAL all the clocks are included, the measure $x_{i,j}$ will produce a nonzero covariance term by coupling to the same measures $x_{i,j,}$ which enters in EAL definition as many times as are the clocks inside or behind the lab i. We obtain:

$$u_{x_j, x_{i,j}} = u_{\left(\sum\limits_{\ell=1}^{N} w_\ell \left(h'_\ell - x_{\ell,j}\right), x_{i,j}\right)} = -u_{\left(\sum\limits_{\ell=1}^{N} w_\ell x_{\ell,j}, x_{i,j}\right)} = -\sum_{\ell=1}^{N_{eq\ lab_i}} w_\ell u_{x_{i,j}}^2 = -W_{eq_i} u_{x_{i,j}}^2 \tag{14}$$

where $N_{eq\ lab_i}$ is the equivalent number of the clocks in the laboratory i, also including the clock external to that lab but that are connected to the UTC through the lab i. We call equivalent weight W_{eq_i} the sum of the $N_{eq\ lab_i}$ clock weights. This will be better explained in the next section, where examples are given. In this case, equation (13) becomes:

$$u_{x_i}^2 = u_{x_j}^2 + u_{x_{i,j}}^2 - 2\ u_{x_{i,j}}^2 W_{eq_i}\ . \tag{15}$$

Equation (15) gives the uncertainty of each clock H_i, given the uncertainty of any clock H_j and the uncertainties of the chains of measures linking clock H_i to clock H_j.

1.3 CORRELATIONS AND ANTICORRELATIONS

In this work we have considered the measures $x_{i,j}$ affected by an uncertainty that we named "uncertainty of the link," which is the one reported in BIPM *Circular T* (Sec. 6). Any time a link $x_{i,j}$ appears in a multiple link such as $x_{i,k} = x_{i,j} + x_{j,k,}$ it gives raise to a correlation term with respect to $x_{i,j,}$ and this is the meaning of the last covariance term in (11).

A more refined evaluation should consider that in case of TWSTFT, GPS/Glonass CV, or Melting Pot, also termed or All-in-View **[8,9]**; the uncertainty of the link is due to different contributions whose correlation properties may differ. For example, in same cases the uncertainty is dominated by systematic calibration uncertainties between lab i and lab j, while in other cases the link uncertainty reported in BIPM *Circular T* is mostly due to the noise of the receivers in both sites.

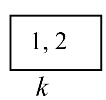
Correlations will always occur in situations wherein the same receiver or system is used to link between two different external labs. The analysis of these effects requires more details than are readily available about correlation of the links. Further study is in progress and a preliminary evaluation indicates that the final uncertainty would change by a very small extent. Some details are reported in the last section devoted to future work.

In this work, we will assume that the noise affecting different nonoverlapping links is uncorrelated, and that correlation will appear only when the same intermediate link appears in multiple links, as will be shown in the example below.

1.4 EXAMPLES OF APPLICATION OF THE METHOD

In order to illustrate the applied procedure, we provide some illustrative examples. In every example, one can see that the uncertainty of each clock H_k with respect to UTC increases as the noise of the lab k link increases, but also decreases as the weight of the clock increases.

Case A1: Let's consider only two clocks (labelled 1 and 2) in the same lab k with no external links, as depicted in the sketch below:



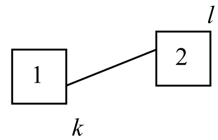
The ensemble time similar to UTC would be computed with these two clocks, and let's suppose clock 1 realizes *UTC (k)*. The uncertainty of $x_k = [UTC - UTC\,(k)]$, by means of equation (11), would be:

$$u^2_{UTC-UTC(k)} = u^2_{UTC-h_1} = w_2^2\, u^2_{x_{1,2}}\,. \tag{16}$$

The uncertainty of the measure $x_{1,2}$ was considered negligible because the clocks are within the same laboratory. Thus

$$u^2_{UTC-UTC(k)} \approx 0\,. \tag{17}$$

Case A2: Let's consider again two clocks (labelled 1 and 2), but maintained in two different labs (k and l, respectively) connected by one measurement link $x_{k,l}$:

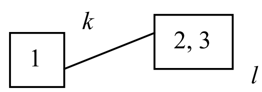


In this case, the uncertainty of the measurement link would be non-negligible and the final uncertainty for *UTC (k),* realized in lab *k* with clock number 1, would be

$$u^2_{UTC-UTC(k)} = w_2^2 u^2_{x_{l,k}} = (1-w_1)^2 u^2_{x_{l,k}}. \tag{18}$$

From (18) we can see that increasing the weight of clock 1 from lab *k* decreases the uncertainty of *UTC – UTC (k).*

Case B1: Let's consider three clocks (labelled 1, 2, and 3), maintained in two different labs (*k* and *l*) connected by a measurement link $x_{k,l}$ according to the sketch below:

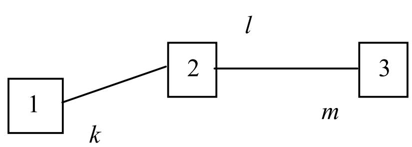


For site *k*, with *UTC (k)* realized by clock 1, the uncertainty on *UTC – UTC (k)* would be*:*

$$\begin{aligned} u^2_{UTC-UTC(k)} &= w_2^2 u^2_{x_{2,k}} + w_3^2 u^2_{x_{3,k}} + 2w_2 w_3 u_{(x_{2,k},x_{3,k})} \\ &= (w_2+w_3)^2 u^2_{x_{l,k}} = (1-w_1)^2 u^2_{x_{l,k}} \end{aligned} \tag{19}$$

since $u^2_{2,k} = u^2_{3,k} = u_{(x_{2,k},x_{3,k})} = u^2_{l,k}$, because clocks 2 and 3 are in the same laboratory.

Case B2: Let's consider three clocks (labelled 1, 2, and 3), located in three different labs (*k, l*, and *m*). We are interested in the uncertainty of *UTC (l)* realized with clock 2 inside the lab *l*, and at first assume the links between lab (*l,k*) and lab (*l,m*) are uncorrelated:



Using (11), where the double summation is zero since the links are not correlated, we obtain,

$$u^2_{UTC-UTC(l)} = w_1^2 u^2_{x_{k,l}} + w_3^2 u^2_{x_{m,l}} \tag{20}$$

For the case where the links between lab (*l,k*) and lab (*l,m*) were correlated through site-based noise in the receiver in lab *l,* but the total noise in both links was the same and equally contributed by the two labs, we would have $u_{(x_{l,k},x_{l,m})} = .5u^2_{l,k} = .5u^2_{l,m}$ and

$$\begin{aligned} u^2_{UTC-UTC(l)} &= w_1^2 u^2_{x_{l,k}} + w_3^2 u^2_{x_{l,m}} + 2w_1 w_3 u_{(x_{l,k},x_{l,m})} \\ &= (w_1^2 + w_1 w_3 + w_3^2)\ u^2_{x_{l,k}} \end{aligned} \tag{21}$$

Case B3: Let's consider three clocks (labelled 1, 2, and 3), located in three labs (*k*, *l* and *m*), topology as for Case B2 above. The uncertainty is evaluated for lab *k* and lab *l* acts as "pivot" to connect lab *m* to lab *k*. The clocks in lab *m* are connected to lab *k* by a double link $x_{k,m} = x_{k,l} + x_{l,m}$. We can see that the noise of the link $x_{k,l}$ affects either the measures of clocks of lab *l* or clocks of lab *m,* and this noise correlates the clock estimates.

Assuming the two links $x_{k,l}$ and $x_{l,m}$ are uncorrelated we obtain, from (11):

$$\begin{aligned} u^2_{UTC-UTC(k)} &= w_2^2 u^2_{x_{l,k}} + w_3^2 \left[u^2_{x_{l,k}} + u^2_{x_{m,l}} \right] + 2 w_2 w_3 u_{(x_{l,k}, x_{m,k})} \\ &= (w_2 + w_3)^2 u^2_{x_{l,k}} + w_3^2 u^2_{x_{m,l}} \end{aligned} \tag{22}$$

where the correlation between $x_{k,l}$ and $x_{k,m}$ is due the common path (*l,k*):

$$u_{(x_{l,k}, x_{m,k})} = u^2_{x_{l,k}} . \tag{23}$$

This example shows that, for uncorrelated links, the weight of labs behind a pivot lab is added to the weight of the pivot lab itself. We call "equivalent weight" W_{eq_i} the weight of the pivot laboratory *i,* which includes also the weights of the clocks behind that pivot lab. W_{eq_i} was also introduced in (14). The last term in (22) takes into account the noise from the pivot (lab *l*) to the remote lab *m*, weighted by the weight of clocks in lab *m*.

If we consider the case of links correlated through site-based noise, the equations leading to (21) would apply, and:

$$u^2_{UTC-UTC(k)} = (w_2^2 + w_2 w_3 + w_3^2)\ u^2_{x_{1,2}} \tag{24}$$

For links dominated by site-based noise, this would be expected, since the dominant noise of that site implies that all ways to combine the external links would yield the same results.

1.5 APPLICATION TO THE UTC COMPUTATION

Starting from the example above, we evaluated the uncertainty corresponding to the network of links currently used for the computation of UTC and reported in Fig. 1.

From Figure 1 we can see that USNO, NIST, NICT[1], and NTSC[2] act as intermediate pivots and that all the measurements are referenced in the end to the PTB laboratory.

[1] NICT is the new name of CLR lab.
[2] All the acronyms appearing in this paper are in agreement with the list reported in BIPM Time Section Annual Reports.

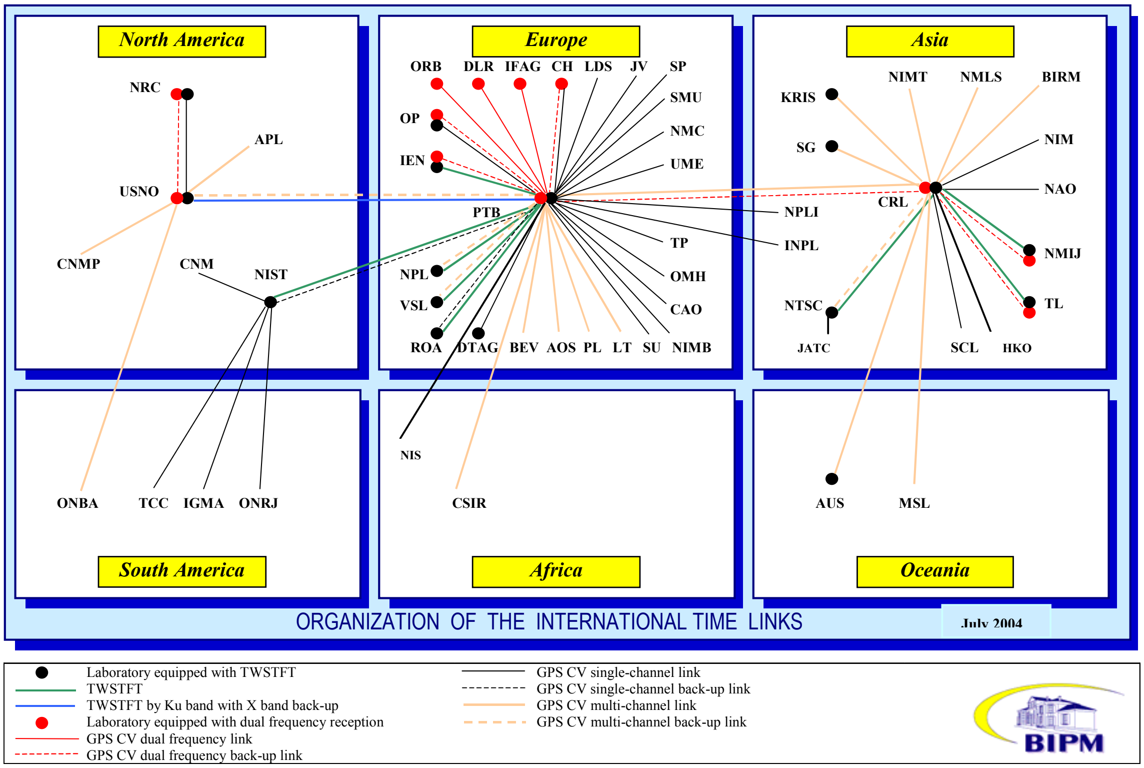


Figure 1. International time links (July 2004).

Since the PTB plays a central role, we can use (11) to obtain the uncertainty for $[UTC - UTC\,(\text{PTB})]$ considering the "equivalent weights" for the intermediate pivot laboratories USNO, NIST, NICT, and NTSC. The final expression turns out to be:

$$
\begin{aligned}
u^2_{UTC-UTC(PTB)} &= \sum_{i=2}^{N_1+1} W_i^2 u^2_{x_{i,UTC(PTB)}} + \sum_{k=N_1+2}^{(N_1+N_2)+1} W_k^2 u^2_{x_{k,UTC(USNO)}} + \sum_{k=(N_1+N_2)+2}^{(N_1+N_2+N_3)+1} W_k^2 u^2_{x_{k,UTC(NIST)}} + \\
&+ \sum_{k=(N_1+N_2+N_3)+2}^{(N_1+N_2+N_3+N_4)+1} W_k^2 u^2_{x_{k,UTC(NICT)}} + W^2_{eq_{USNO}} u^2_{x_{UTC(PTB),UTC(USNO)}} + W^2_{eq_{NIST}} u^2_{x_{UTC(PTB),UTC(NIST)}} + \\
&+ W^2_{eq_{NICT}} u^2_{x_{UTC(PTB),UTC(NICT)}} + W^2_{eq_{NTSC}} u^2_{x_{UTC(PTB),UTC(NTSC)}} + W^2_{JACT} u^2_{x_{UTC(NTSC),UTC(JACT)}}
\end{aligned} \tag{25}
$$

Here $N_1 = 25$ is the number of laboratories directly linked to PTB with one single link (IEN, AOS, SP, LT, etc.), excluding the four labs acting as intermediate pivots; $N_2 = 4$ is the number of laboratories linked to USNO (namely NRC, CNMP, APL, ONBA); $N_3 = 4$ is the number of laboratories linked to NIST (namely TCC, ONRJ, CNM, IGMA); $N_4 = 13$ is the number of laboratories linked to NICT (SG, TL, NAO, MSL, etc.); W_{eq_i} are "equivalent weights" of laboratories as introduced in (14).

The "equivalent weight" of the USNO is, for example:

$$W_{eq_{USNO}} = W_{NRC} + W_{CNMP} + W_{APL} + W_{ONBA} + W_{USNO}\,.$$

Similar expressions hold for the other intermediate pivots, NIST, NICT, and NTSC, and W_{lab} was defined in (12).

Once the uncertainty of $[UTC - UTC\,(\text{PTB})]$ is evaluated, the expression (15) can be used to compute the uncertainty of $[UTC - UTC\,(k)]$ for every other laboratory k.

2 A MONTE CARLO SIMULATION

The analytical approach presented in the previous section was tested by a Monte Carlo simulation using the BIPM's software ALGOS, on the full set of clock and time transfer data used to generate the *Circular T* of July 2004. For that month, the pivot laboratories were PTB, USNO, NIST, NICT, and NTSC.

Since the source of uncertainty in *UTC – UTC (k)* are the links whose uncertainty $u^2_{x_{i,j}}$ is listed in the BIPM *Circular T*, we performed a simulation by assuming that every link measure is described by a random variable with Gaussian distribution, mean value equal to the obtained measurement value *a*, and standard deviation equal to $u^2_{x_{i,j}}$.

The Monte Carlo simulation consists in picking up different values for the measure $x_{i,j}$ coming from its statistical distribution and computing *UTC – UTC (k)* with the simulated measure values. This gives an indication of the variability of the results *UTC – UTC (k)* due to the variability of the measures $x_{i,j}$, which are, in our assumption, the only source of variability and, hence, of uncertainty.

We proceed in the simulation step by step firstly evaluating the effect on *UTC – UTC (k)* of only one noisy link (all the other links are considered with negligible noise); then the noise is inserted link by link and we compare this obtained "experimental" variability with respect to the expected theoretical results coming from the analytical estimation (11).

2.1 ONE LINK

We began by studying the effect of the uncertainty of the link between the PTB and the LT labs using the data of July 2004. Laboratory LT has one clock of percentage weight $w_{LT} = 0.119$; the uncertainty of the link [*UTC* (LT) – *UTC* (PTB)] is $u^2_{UTC(LT)-UTC(PTB)} = 27.25\, ns^2$, and on MJD 53144 [*UTC* (LT) – *UTC* (PTB)] measurement value was equal to –242.8 ns. We generated a large number *N* of experimental values of [*UTC* (LT) – *UTC* (PTB)], normally distributed around the central value [*UTC* (LT) – *UTC* (PTB)] = –242.8 ns and with a variance of $u^2_{UTC(LT)-UTC(PTB)} = 27.25\, ns^2$.

- Theory

Using (11), with only the link (*PTB,LT)* corrupted with noise, we have

$$u^2_{UTC-UTC(PTB)} = w^2_{LT} u^2_{UTC(LT)-UTC(PTB)}$$
$$= \left(\frac{0.119}{100}\right)^2 (27.25) = 3.85x10^{-5}\, ns^2$$

Thus

$$u_{UTC-UTC(PTB)} = 0.0062\, ns,$$

and from (17)

$$u_{UTC-UTC(LT)} = 5.21\, ns.$$

Alternatively, using equation (11) directly referenced to *LT* lab, which correspond to having the total clock weight in PTB with the exception of the LT clock, we obtain the same theoretical result:

$$u_{UTC-UTC(LT)} = \left(1 - \frac{0.119}{100}\right) \cdot \sqrt{27.25} = 5.21\ ns\ .$$

- <u>Experimental Results</u>

Using N = 10 000 simulated measurement values, we observe a normal distribution of the obtained values [*UTC* – *UTC* (PTB)] with a mean value equal to –16.7087 ns and standard deviation equal to 0.0063 ns; the value [*UTC* – *UTC* (PTB)] for that date published in *Circular T* was in fact equal to –16.7 ns. Therefore, the mean value and the standard deviation of *UTC* – *UTC* (PTB) obtained by simulating the noisy link *PTB,LT* correspond to the published value and to the expected theoretical uncertainty.

For the laboratory *LT* we obtained a normal distribution of mean value equal to 226.161 ns with a standard deviation of 5.2 ns. The published value in *Circular T* was [*UTC* – *UTC* (LT)] = 226.1 ns.

The same results are depicted in the figures below, where the histogram reports the obtained results from the Monte Carlo simulation, while the solid line (in red) represent the expected statistical distribution of the values according to the theoretical analysis.

The analytical and simulation results agree perfectly.

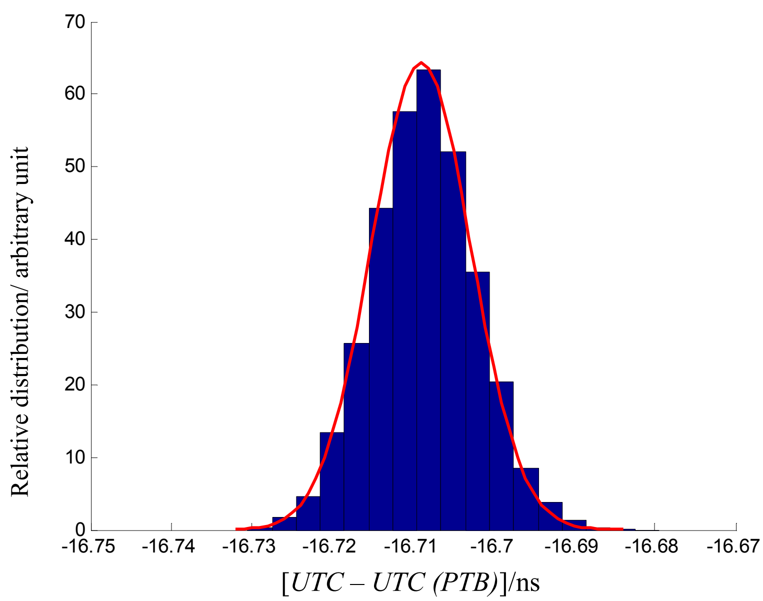


Figure 2. Theoretical and experimental distribution of the values *UTC – UTC (PTB)* considering only one link with noise.

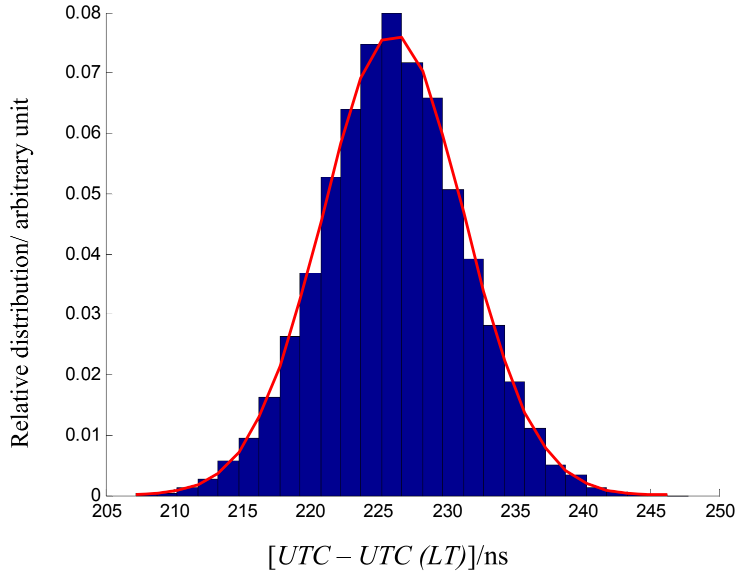


Figure 3. Theoretical and experimental distribution of the values *UTC – UTC (LT)* considering only one link with noise.

2.2 COMPLETE SYSTEM

The Monte Carlo results are here reported for the complete system of links, using N = 20 000 simulated data for any measure. Figure 4 shows the close agreement for example between the theoretical and experimental results for the values of [*UTC – UTC* (NIST)]. In Table 1, we can see the analytical compared to the simulation results obtained considering every links with noise. The agreement between the two estimation methods is quite impressing giving complete confidence on the analytical development.

3 FUTURE EXTENSION

To refine the evaluation of uncertainties, it is necessary to know more details than are readily available about the correlation of the links. This is largely because, as noted in the examples, the correlated noise between two sites should include the contribution from site-based errors at sites along the path of links and because the correlation between two links with a common site that uses the same equipment for both links should include the site-based noise contribution from that site. This is difficult to estimate when calibrations are done by links rather than by sites. In our computations, we have used the uncertainties as reported in the *Circular T*. We have supplemented these by more exact calculations, taking the above considerations into account as best we could, and found that on the whole the effects of these simplifications are small.

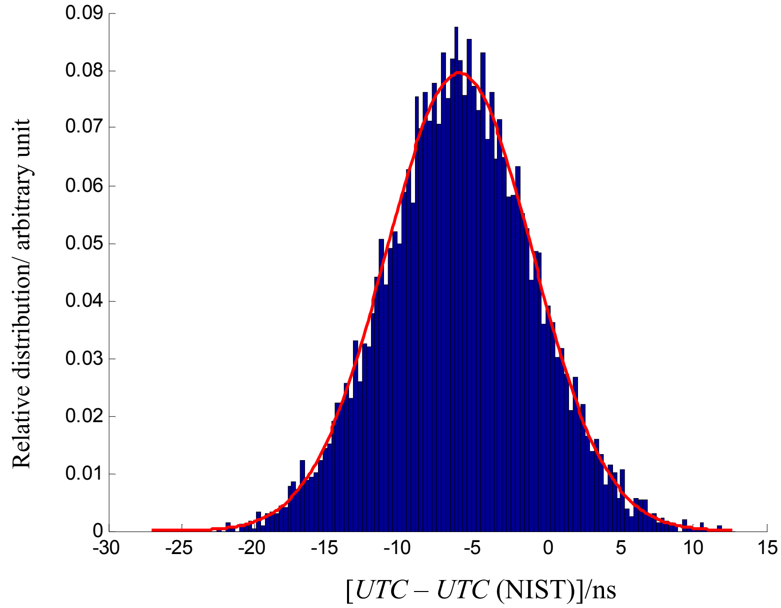


Figure 4. Theoretical and experimental distribution of the values of [*UTC* – *UTC* (NIST)].

Further extension of this evaluation would be based on the following consideration. If all time transfer were achieved using a single system per site, and if all sources of noise were site-based, such as mostly happens for Melting Pot GPS, also termed All-in-View (AV) **[8,9]**, then all possible links would obey the following closure relation:

$$x_{i,j}(t)+x_{j,k}(t)+x_{k,i}(t)=0 \tag{26}$$

In this situation, the noise of each site's time transfer system would be indistinguishable from the noise of its clocks and the dominant uncertainty would be given by the site noise and all the external clock would be seen through that dominant noise independently on their location, but as they were all inside a single external lab as in the example B1 reported above. This noise would affect *UTC – UTC (k)* itself according to the total weight of the site clocks, but due to the closure relation it would not affect the difference between the clocks of any other two sites. Therefore, in such a situation the uncertainty of the site *k* affecting *UTC – UTC (k)* would be written according to (21):

$$u^2_{UTC-UTC(k)}=(1-w_k)^2\,u^2_{k,k} \tag{27}$$

where $u_{k,k}$ is the uncertainty of site *k*'s time transfer system.

If different satellite schedules are used in the relevant links, uncertainties in time transfer using GPS common view (CV) are largely, though not entirely, laboratory-based. Even for CV observations made using every available satellite, closure violations will only arise if simultaneous satellite observations are recorded at only two of the three sites. In such cases, orbit mis-estimation and receiver noise will contribute uncertainties, and any azimuth or elevation-dependent asymmetries in the multipath environment would cause both uncertainties and biases. Since calibration is achieved by an all-sky sampling that is systematically different from the sky-sampling of CV, systematic multipath will also lead to uncertainties. Despite these noise sources, the closure relation largely holds for common view, and the largest source of uncertainty is typically due to variations of the receiver system that are common to all the data.

Table 1. Analytical and numerically estimated uncertainties of all the UTC participating laboratories, considering every link with noise.

k	$u[UTC - UTC\,(k)]$/ns		k	$u[UTC - UTC\,(k)]$/ns	
	Analytical method	Numerical method		Analytical method	Numerical method
AOS	5.6	5.6	NIS	20.3	20.6
APL	5.7	5.7	NIST	5.0	5.0
AUS	7.3	7.3	NMC	20.7	20.8
BEV	5.5	5.5	NMIJ	7.0	7.0
BIRM	20.6	20.6	NPL	5.3	5.2
CAO	21.2	21.0	NPLI	20.3	20.4
CH	5.4	5.3	NRC	15.2	15.1
CNM	21.0	21.1	NTSC	7.1	7.0
CNMP	8.4	8.4	OMH	20.2	20.5
CSIR	20.3	20.6	ONBA	8.9	8.9
DLR	5.4	5.4	ONRJ	21.2	21.3
DTAG	10.6	10.5	OP	5.4	5.4
HKO	7.3	7.3	ORB	5.3	5.4
IEN	2.4	2.4	PL	5.4	5.3
IFAG	5.3	5.3	PTB	1.9	1.9
IGMA	20.8	20.9	ROA	5.4	5.4
INPL	10.9	11.0	SCL	11.6	11.6
JATC	21.2	21.3	SG	20.5	20.6
JV	20.7	20.6	SMU	20.7	20.7
KRIS	7.1	7.2	SP	10.4	10.5
LDS	20.3	20.2	SU	6.1	6.1
LT	5.6	5.6	TCC	21.2	20.9
MSL	20.7	20.8	TL	6.7	6.7
NAO	20.6	20.7	TP	5.8	5.8
NICT	4.4	4.4	UME	25.1	25.1
NIM	20.4	20.3	USNO	2.3	2.3
NIMT	20.7	20.9	VSL	5.3	5.3

For Two-Way Satellite Time and Frequency Transfer (TWSTFT), the noise is again largely site-dependent. Some closure violations can occur because the observations between pairs of sites are typically made at different codes and slightly different frequencies. The largest source of closure errors is probably due to the fact that the received signals are shaped by the product of the transmitting and receiving bandpasses, while the delay and certain noise components such as the cable-dependent multipath can systematically vary over the bandpass **[10]**. While TWSTFT closure violations are seen at the 1-ns level in the data sent to the BIPM, they could be reduced through baseline-dependent calibrations. In those cases where a TWSTFT system is calibrated with GPS, the uncertainties in the calibration are determined by the uncertainties in the GPS calibration.

Special situations arise when one site is a pivot site, connected to some sites by one technique and other sites by a different technique. By means of illustration, we can consider a very simplified situation in which every site is directly linked to one central pivot site, either by AV GPS or TWSTF; this can easily be generalized to describe more complex topologies. We will assume variations between any two links are completely uncorrelated with respect to variations at the link extremities, but that the links are 100% correlated with respect to variations of the equipment at the central pivot site, provided both links are by either TWSTFT or by GPS. Let us also assume a bias B exists in the

GPS equipment at the pivot site. In this case, it is easy to show that the bias would affect those laboratories linked by GPS to the pivot as follows:

$$\Delta_{UTC-UTC(k)} = (1 - W_G)B$$

where W_G is the sum of all the weights of the laboratories linked to the pivot site by GPS.

The bias would affect the pivot laboratory and those linked to it by TWSTFT as follows:

$$\Delta_{UTC-UTC(k)} = -W_G \; B$$

Under the normal circumstances described in this paper, the existence of any biases would not carry any significant statistical implications, as they would be directly related to the tabulated uncertainties in the links themselves. However, the above equations illustrate the dependence of TAI and UTC upon the equipment at any central pivot, which may not always follow the Gaussian behavior assumed in this work, particularly in the case of equipment failure.

4 CONCLUSIONS

This paper presents a preliminary study of the determination of the uncertainties of [*UTC – UTC (k)*]. An analytical solution is derived from the law of the propagation of uncertainty, taking into account that leap seconds and deterministic frequency steering of EAL do not affect these uncertainties. The analytical results were verified through Monte Carlo simulations using the software used to generate UTC, and a perfect agreement was found, giving complete confidence on the analytical estimation.

A more detailed analysis is in progress, including full inclusion of all available calibration information, more details for the correlation between the links, methods for optimizing the link structure, given uncertainty information, non-Gaussian behavior, and different correlation properties of uncertainties due to calibration or due to random noise.

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